Recursion and the Determinant of a Matrix

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Overview

Our goal this week is to explore some examples of recursive function definitions and to write a recursive function to compute the determinant of a square matrix.

Activities

Recursion:
The process of solving a problem by reducing it to smaller instances of itself is called recursion.

- Compute $1 + 2 + 3 + \cdots + n$:

  ```
  function s = mysum(n)
      if(n == 1)
          s = 1;
          return;
      end
      s = n + mysum(n - 1);
  end
  ```

- Compute the nth Fibonacci number:

  ```
  function fibn = myfib(n)
      if((n == 1) || (n == 2))
          fibn = 1;
          return;
      end
      fibn = myfib(n - 1) + myfib(n - 2);
  end
  ```

- Tower of Hanoi:
  You are given three columns, the first of which contains n rings, stacked with decreasing size. The object of this game is to move all the rings from the first column to the third column. The rings should be moved one by one, and a ring cannot be placed on top of a smaller ring.

  Play it online: http://www.novelgames.com/flashgames/game.php?id=31

  Game strategy: The trick to move a large pile of n pieces is to move the first $n - 1$ pieces to another rod, then move the last piece to the destination rod, and then move the $n - 1$ pieces to the destination rod.

MATLAB code:

```
% This program prints the sequence of moves
% needed to transfer the n disks from the
% "from" peg to the "dest" peg by using the
% "by" peg.

function hanoi(n, from, by, dest)
    if(n == 1)
        fprintf('\t Move disk 1 from peg ');
        fprintf('%c to peg %c.\n', from, dest);
        return;
    end
    hanoi(n - 1, from, dest, by);
    fprintf('\t Move disk %d from peg ', n);
    fprintf('%c to peg %c.\n', from, dest);
    hanoi(n - 1, by, from, dest);
end
```

Assignment

Submit your M-file and a diary that shows how you tested the code.

Create an M-file *mydet.m* to compute the determinant of a square matrix using recursion. The function should take as input a matrix A and return its determinant d. Test your code on the following matrix and use **det(·)** to check your answer.

$$B = \begin{bmatrix} -7 & 4 & -2 & -8 & 6 \\ 8 & 7 & 2 & 3 & -1 \\ 6 & -6 & 6 & 0 & -7 \\ -6 & 2 & -9 & 2 & 0 \\ -9 & 6 & 7 & 5 & 0 \end{bmatrix}$$

Hint: Computing the determinant using recursion:
The determinant of an $n \times n$ square matrix A can be calculated in the following way:

$$\det(A) = a_{11}C_{11} + a_{12}C_{12} + \cdots + a_{1n}C_{1n}$$

where $C_{1j} = (-1)^{1+j}\det(M_{1j})$ and the $(n-1) \times (n-1)$ submatrix M_{1j} of A throws out row 1 and column j.

Please note that next week, April 8th, we will take lab quiz #2.