Least Squares Approximation

Toby Sanders (sandertl@mailbox.sc.edu)
Department of Mathematics

Overview

Given observed data, we compute the least squares approximation line and plot the results using MATLAB.

New (and old) MATLAB commands

- **Saving and Loading Data in MATLAB:**

  ```
  >> save A
  ```

  This saves the variable A as *A.mat*, a binary file.

  ```
  >> save('my_data', 'A', 'v', 'b')
  ```

  This saves the variables A, v and b into the file *my_data.mat.*

  ```
  >> load A
  ```

  This loads the variables saved in *A.mat* to the workspace.

  ```
  >> B = load('A.mat');
  ```

  This loads the variables saved in *A.mat* to the workspace, into a structure array named B.

- **Slash or matrix right division:**
  B/A is roughly the same as $B \times A^{-1}$.

- **Backslash or matrix left division:**
  $A\backslash B$ is roughly the same as $A^{-1} \times B$.

- **Plotting**

  ```
  >> plot(x,y,'r',u,v,'b')
  ```

  Assuming x, y and u, v are vectors of the same dimension, this command plots x, y together in red and u, v together in blue as sets of ordered pairs.

Least Squares Approximation

For the points $(1,5)$, $(2,5)$, $(7,8)$, $(8,7)$, find the "best-fit" linear function to approximate the data and plot the resulting line along with the data points.
Set up: Ultimately, we are looking for a line $y = mx + b$ that models the data, i.e. we want to find an approximate solution to the following system:

$$\begin{cases} m \cdot 1 + b &= 5 \\ m \cdot 2 + b &= 5 \\ m \cdot 7 + b &= 8 \\ m \cdot 8 + b &= 7 \end{cases}$$

We proceed by setting

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 7 & 1 \\ 8 & 1 \end{bmatrix}, \qquad X = \begin{bmatrix} m \\ b \end{bmatrix}, \qquad B = \begin{bmatrix} 5 \\ 5 \\ 8 \\ 7 \end{bmatrix},$$

and solving $A^T A\hat{X} = A^T B$ for $\hat{X}$, which is the least squares approximation for X.

```
>> points = [1 5; 2 5; 7 8; 8 7]
>> A = ...
>> ...
>> B = ...
>> ...
>> X = (A'*A)\(A'*B)
>> m = X(1)
>> b = X(2)
>> y = m*points(:,1) + b
>> plot(points(:,1), points(:,2), 'r*', points(:,1), y, 'b');
>> legend('Real Values', 'Approximation');
>> xlabel('x');
>> ylabel('y');
```