LU FACTORIZATION

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OVERVIEW

We develop a MATLAB implementation (M-file) of LU factorization for a matrix A (in the case that elimination can be performed without row exchanges).

NEW STRUCTURE IN MATLAB – CELLS

MATLAB has the ability to store many types of variables. One such is call a cell structure. A cell is like a matrix, but each entry can be a variable of its own, such as a matrix, character, another cell. For example,

```
>>T = cell(2,3);
>>T{1,1}=rand(5,2);
>>T{1,2}=sin(1);
>>T{2,3}='Wowowow';
```

Accessing entries of a cell is similar to a matrix, but use {} brackets instead of ().

```
>>T{2,3}
```

ACTIVITIES

We will perform the LU factorization of the matrix

$$A = \begin{bmatrix} 8 & 0 & 0 & 1 \\ 8 & 7 & 4 & 8 \\ 4 & 4 & 5 & 9 \\ -4 & -1 & 2 & 2 \end{bmatrix}.$$

To initialize L and U, type

```
>> L = eye(4)      % L is the 4 by 4 identity matrix initially
>> U = A           % U is A initially
```

To perform elimination on the first column, type

```
>> L(2,1) = U(2,1)/U(1,1)               % Here we are forcing
>> U(2,1:4) = U(2,1:4)-L(2,1)*U(1,1:4)  % U(2,1), U(3,1), and U(4,1)
>> L(3,1) = U(3,1)/U(1,1)               % to be 0. Why are we
>> U(3,1:4) = U(3,1:4)-L(3,1)*U(1,1:4)  % performing valid
>> L(4,1) = U(4,1)/U(1,1)               % computations?
>> U(4,1:4) = U(4,1:4)-L(4,1)*U(1,1:4)
```

To perform elimination on the second column, type

```
>> L(3,2) = U(3,2)/U(2,2)              % Here we are forcing
>> U(3,2:4) = U(3,2:4)-L(3,2)*U(2,2:4) % U(3,2) and U(4,2)
>> L(4,2) = U(4,2)/U(2,2)              % to be 0.
>> U(4,2:4) = U(4,2:4)-L(4,2)*U(2,2:4)
```

Note here that we do not need the entries in the first column (or first row). Why?
   To perform elimination on the third column, type

```
>> L(4,3) = U(4,3)/U(3,3)              % Why are these lines okay?
>> U(4,3:4) = U(4,3:4)-L(4,3)*U(3,3:4)
```

Note here that we do not need the entries in the first two columns (or first two rows).
Why? You now have an LU factorization of A. To check if it is correct, type

```
>> L*U
```

ASSIGNMENT

Submit your M-files and a diary showing that you called on these functions as suggested.
   1. Finish the following M-file. Run your function on the same matrix A as given
above.

```
% LU4 - The function in this M-file computes an LU factorization
% of a 4 x 4 matrix under the assumption that elimination can be
% performed without row exchanges.
% Input: 4 x 4 matrix A;
% Output: lower triangular matrix L and upper triangular matrix U.

function [L,U] = LU4(A)
    L = eye(4);
    U = A;
    for j= ...
        for i= ...
            L(...) = ...; % Do not forget the semicolons here
            U(...) = ...; % to suppress intermediate output!
        end
    end
end
```

   2. Write a function **MYLU** to perform LU factorization for an arbitrary $n{\times}n$ matrix
(under the assumption that elimination can be performed without row exchanges). Run
the function on the input

$$
T = \begin{bmatrix}
-3 & -9 & -5 & -7 & -1 \\
-8 & 8 & -6 & 0 & 5 \\
2 & 2 & -4 & 2 & -4 \\
1 & 7 & 0 & 4 & -6 \\
4 & 2 & 7 & 5 & 5
\end{bmatrix}.
$$